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QUANTUM-MECHANICAL HAMILTONIAN OF THE NUCLEAR-LIKE STATES IN THE SU(2)-SKYRME MODEL

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Quantum mechanical Hamiltonian for the nuclear-like states in the SU(2)-Skyrme model has been obtained in the framework of the collective coordinate method. The results of the calculation of the rotational bands are also presented with and without taking into account vibrational degrees of freedom.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Квантово-механический гамильтониан ядерноподобных состояний в SU(2)-модели Скирма

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В рамках метода коллективных координат получен квантово-механический гамильтониан ядерноподобных состояний SU(2)-модели Скирма. Представлены результаты расчета ротационных полос с учетом и без учета вибрационных степеней свободы.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

In [1] non-toroidal skyrmions up to baryon number $B \leq 12$ have been investigated at classical level. A generalized ansatz for the variational form of the chiral field U:

$$U(\vec{r}) = cosF(r) + \left(\vec{\tau} \cdot \vec{N}\right) sinF(r) , \tag{1}$$

has been used. Isotopic vector $\vec{N}$ determines the considered configurations:

$$\vec{N} = \left\{ \begin{array}{c} sinT(\theta)\ cos\Phi(\theta\ ,\phi) \\ sinT(\theta)\ sin\Phi(\theta\ ,\phi) \\ cosT(\theta) \end{array} \right\} , \tag{2}$$

where $\Phi(\theta,\phi)$, $T(\theta)$ are some arbitrary functions of angles (θ,ϕ) of the vector $\vec{r}$ in the spherical coordinate system. With such form of the chiral field a number of solitons with non - toroidal baryon charge density distribution has been obtained. Their masses are depending linearly on the baryon charge and are smaller than the masses of the toroidal solitons with the same charge. For example, a classical soliton composed from two toroidal dibaryons has a mass smaller than the mass of one toroidal soliton with baryon charge four.

We point out in [1] that among the quantum states of the toroidal skyrmions there are none with quantum numbers of the triton ground state. Although the quantum state that corresponds to a nontoroidal configuration has a chance to obtain "right" quantum numbers and to become bounded after quantization.

In the present paper we calculate the quantum mechanical Hamiltonian for a class of nuclears like states. We try to show that the very restrict connection between the third components of the spin and isospin in body fixed system may be removed for the considered space of states.

Let us use the collective coordinate method for the class of the solutions described in [1]. We will consider only field configurations with symmetrical baryon charge density distributions relative to the (x,y) plane. More formally it means, that we consider only solutions $\left(n,l,\{k\}_1^l\right)$ which obey the conditions:

$$\begin{cases} k_i = k_{l+1-i}\ , & i = 1,\ldots\frac{l-1}{2}\ , \quad for\ odd\ l, \\ k_i = -k_{l+1-i}\ , & i = 1,\ldots\frac{l}{2}\ , \quad for\ even\ l. \end{cases} \tag{3}$$

These symmetrical solutions include almost all configurations we are interested in. Namely for these configurations the classical masses are linearly depending on the baryon charge.

We have to make some remarks about the scale variable λ for the breathing mode. Generally one may introduce a number of λ's: one for each region of space, labeled by the angles θ^i. But in such case we will have to take into account the center of mass motion when we introduce rotations. For simplicity and as a first approximation let us introduce only one scaling parameter λ. The way one can introduce the vibrational and rotational coordinates is well known [2], [3], [4], [5]. These coordinates transform the stationary solution

$$U_0(\vec{x}) = \sigma(\vec{x}) + i\left(\vec{\tau}\cdot\vec{\phi}(\vec{x})\right) \tag{4}$$

by the following way:

$$U_0(\vec{x}) \rightarrow U(\vec{x},t) = \sigma(\vec{x}') + i\tau^i I^{ij}\phi^j(\vec{x}')\ , \tag{5}$$

where

$$x'_n(t) = e^{\lambda(t)} \left(R^{-1}\right)_{nm} x_m \ . \tag{6}$$

Here $I(t)$ and $R(t)$ are global orthogonal 3×3 -matrices. Matrix I describes the isorotations, and R - the space rotations. Their parameters serve as collective coordinates. The frequences of the isotopic ω^k and space Ω_k rotations are ordinarily introduced as

$$\dot{I}^{ik} \left(I^{-1}\right)^{kj} = \epsilon^{ijk}\omega^k \quad , \overline{\omega}^k = I^{kj}\omega^j \ ,$$
$$\left(\dot{R}^{-1}\right)_{ik} R_{kj} = -\epsilon_{ijk}\Omega_k \ . \tag{7}$$

The part of the Lagrangian depending on the rotational coordinates now is

$$L^{rot} = \sum_{m=1}^{l} L^{rot}_m = \frac{1}{2}\sum_{m=1}^{l}\Big[Q^m_S\left(\Omega_1^2+\Omega_2^2\right) +$$
$$+ \quad Q^m_T\left(\overline{\omega}_1^2+\overline{\omega}_2^2\right) + Q^m\left(\overline{\omega}_3^2 - 2k_m\overline{\omega}_3\Omega_3 + k_m^2\Omega_3^2\right)\Big] \ . \tag{8}$$

In this expression the inertial momenta are of the form:

$$Q^m_T(\lambda) = \frac{\pi}{F_\pi e^3}\int_0^\infty x^2dx \int_{\theta_{m-1}}^{\theta_m} sin\theta d\theta\Big\{-e^{-\lambda}\frac{sin^4F}{x^2}\times$$
$$\times\left(k^2\frac{sin^2T}{sin^2\theta}cos^2T + (T')^2\right) + sin^2F\left[\frac{e^{-3\lambda}}{4} + e^{-\lambda}\left((F')^2 + \right.\right.$$
$$\left.\left. + \left[k^2\frac{sin^2T}{sin^2\theta} + (T')^2\right]\frac{sin^2F}{x^2}\right)\right]\left(1+cos^2T\right)\Big\} \ , \tag{9}$$

$$Q^m_S = \frac{\pi}{F_\pi e^3}\int_0^\infty x^2dx \int_{\theta_{m-1}}^{\theta_m} sin\theta d\theta\Big\{-e^{-\lambda}\frac{sin^4F}{x^2}\times$$
$$\times\left[k^4\frac{sin^4T}{sin^4\theta}cos^2\theta + (T')^4\right] + sin^2F\left[\frac{e^{-3\lambda}}{4} + e^{-\lambda}\left((F')^2 + \right.\right.$$
$$\left.\left. + \left[k^2\frac{sin^2T}{sin^2\theta} + (T')^2\right]\frac{sin^2F}{x^2}\right)\right]\left(k^2\frac{sin^2T}{sin^2\theta}cos^2\theta + (T')^2\right)\Big\} \tag{10}$$

$$Q^m(\lambda) = \frac{2\pi}{F_\pi e^3}\int_0^\infty x^2dx \int_{\theta_{m-1}}^{\theta_m} sin\theta d\theta\Big\{-e^{-\lambda}\frac{sin^4F}{x^2}k^2\frac{sin^4T}{sin^2\theta} + \tag{11}$$

$$sin^2F\left[\frac{e^{-3\lambda}}{4}+e^{-\lambda}\left((F')^2+\left[k^2\frac{sin^2T}{sin^2\theta}+(T')^2\right]\frac{sin^2F}{x^2}\right)\right]sin^2T\right\} .$$

Formulas (8-11) are obtained only for $k \neq 1$ case. The impulses conjugated to the variables $\overline{\omega}^k$, Ω_k are

$$\begin{aligned} S_i^{b.f} &= \sum_{m=1}^{l} Q_S^m(\lambda)\Omega_i \ , \ for\ i=1,2 \ ; \\ S_3^{b.f.} &= \sum_{m=1}^{l} k_m Q^m(\lambda)\left\{k_m\Omega_3-\overline{\omega}^3\right\} \ , \end{aligned} \tag{12}$$

$$\begin{aligned} T_i^{b.f.} &= \sum_{m=1}^{l} Q_T^m(\lambda)\overline{\omega}^i \ , \ for\ i=1,2 \ ; \\ T_3^{b.f.} &= \sum_{m=1}^{l} Q^m(\lambda)\left\{\overline{\omega}^3-k_m\Omega_3\right\} \ . \end{aligned} \tag{13}$$

It is not difficult to obtain the effective mass for the scaling vibrations:

$$\begin{aligned} m(\lambda) &= \frac{2\pi}{F_\pi e^3}\int_0^\infty (F')^2\left\{\frac{e^{-3\lambda}}{2}+e^{-\lambda}\frac{sin^2F}{x^2}\times\right. \\ &\times \left.\int_0^\pi\left[k^2\frac{sin^2T}{sin^2\theta}+(T')^2\right]sin\theta d\theta\right\}x^4dx \ . \end{aligned} \tag{14}$$

In such way we obtain a Hamiltonian of the form

$$\hat{H} = M(\lambda)+\frac{\hat{p}^2}{2m(\lambda)}+\frac{\hat{T}^2}{2Q_T(\lambda)}+\frac{\hat{S}^2}{2Q_S(\lambda)}+H_1 \ , \tag{15}$$

where

$$Q_T(\lambda)=\sum_{m=1}^{l} Q_T^m(\lambda) \ , \quad Q_S(\lambda)=\sum_{m=1}^{l} k_m^2 Q_S^m(\lambda) \ . \tag{16}$$

The depending on the internal quantum variables part H_1 of the Hamiltonian is

$$H_1=\frac{1}{2}\left[\frac{Q_1}{Q_1Q_2-Q_0^2}-\frac{1}{Q_T}\right]\left(\hat{T}_3^{b.f.}\right)^2+$$

$$+\frac{1}{2}\left[\frac{Q_2}{Q_1Q_2-Q_0^2}-\frac{1}{Q_S}\right]\left(\hat{S}_3^{b.f.}\right)^2+\frac{Q_0}{Q_1Q_2-Q_0^2}\hat{T}_3^{b.f.}\hat{S}_3^{b.f.} \quad (17)$$

for odd l and

$$H_1=\frac{1}{2}\left[\frac{1}{Q_2}-\frac{1}{Q_T}\right]\left(\hat{T}_3^{b.f.}\right)^2+\frac{1}{2}\left[\frac{1}{Q_1}-\frac{1}{Q_S}\right]\left(\hat{S}_3^{b.f.}\right)^2 \quad , \quad (18)$$

for even l. In the last expressions

$$Q_1(\lambda)=\sum_{m=1}^{l}k_m^2Q^m(\lambda)\ ,\quad Q_2(\lambda)=\sum_{m=1}^{l}Q^m(\lambda)\ ,$$
$$Q_0(\lambda)=\sum_{m=1}^{l}k_mQ^m(\lambda)\ . \quad (19)$$

It is seen now, that in the general case there is not a connection between the third spin and isospin components in body fixed system as it was for pure toroidal solutions. We have to note that such a coupling holds for the regions (θ_{m-1},θ_m):

$$S_{3,m}^{b.f.}=k_mQ^m\left(k_m\Omega_3-\overline{\omega}_3\right)\quad , \quad T_{3,m}^{b.f.}=-Q^m\left(k_m\Omega_3-\overline{\omega}_3\right)\ . \quad (20)$$

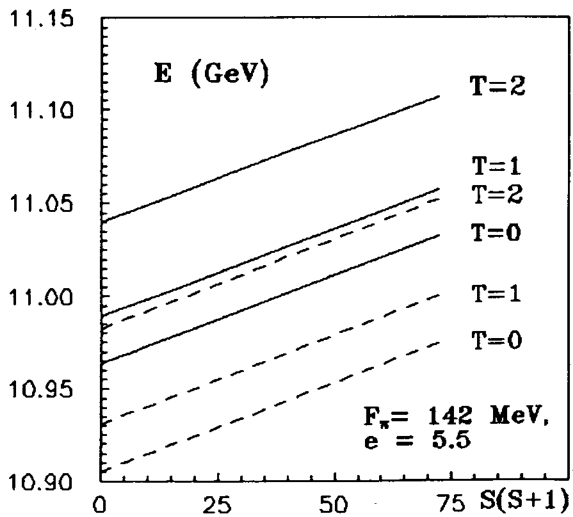

Fig.1 The rotational bands for baryon charge $B = 12$ nuclear - like state.

In Fig.1 one sees the rotational bands calculated in accordance with equation (15). We have to note, that the including of the vibration does not practically change the deflection angles of the rotational bands. This differs the considered nuclear case from the dibaryon one [6]. In Fig.1 the solid line corresponds to the calculations in which the vibrations have been taken into account.

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